MATHEMATICS ANALYSIS AND APPROACHES HIGHER LEVEL EXPLORATION

Exploring the method of calculating the surface area of solid of revolution and estimating the lateral surface area of the vase

WORD COUNT: 3466

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1. Introduction

Figure 1. Photo of the vase (Student's own work)

In my free time, I enjoy refurbishing old, and dull objects that surround me and otherwise would be thrown away. It includes old wooden furniture and a variety of decorations made of different materials. This hobby complies with the idea of a circular system that I developed during my geography lessons which places importance on reusing and recycling things and resources. I am also a provident person who does not like it when something is wasted, therefore I always try to minimize the costs and loss of resources. My next project involves painting a glass vase presented in Figure 1. that, in my opinion, is dull and would look definitely more interesting with a vibrant finish. I opt in using the acrylic enamel which ensures the long-lasting of the final product so that it will serve longer and it complies with my eco-friendly attitude. However, this paint is very expensive and I would like to calculate the surface area of the vase to evaluate whether it is a viable option or I should choose another slightly cheaper alternative. In order to estimate the cost of the project, I will use the paint coverage parameter, that is "amount of paint that will cover a given surface" (Nerolac, 2022). Unfortunately, there is no paint coverage data available for that type of paint. Therefore, I decided to assume the average paint coverage of high-performance acrylic emulsion, which is 9 m^2/l because it has similar nature to the acrylic enamel (Paint Coverage Rates, n.d.). No matter what paint I will decide to use, I hope to discover a method for calculating the lateral surface area of any solid of revolution, that could be useful in the industry on a mass scale where even small differences in cost have a much greater effect.

2. Background information and methodology

For the sake of the investigation, I will assume that my vase is a perfect volume of revolution. Even though it is not in the reality, this approximation will allow me to calculate the surface area which is similar to the real surface area of the vase.

As an extension to the integral calculus, during math HL lessons, we have been familiarized with the concept of calculating the volume of solid produced by the revolution of the function, y about the x-axis in the interval [a, b]. We performed it according to the formula in the booklet, that is:

$$V = \int_a^b \pi y^2 \, dx$$

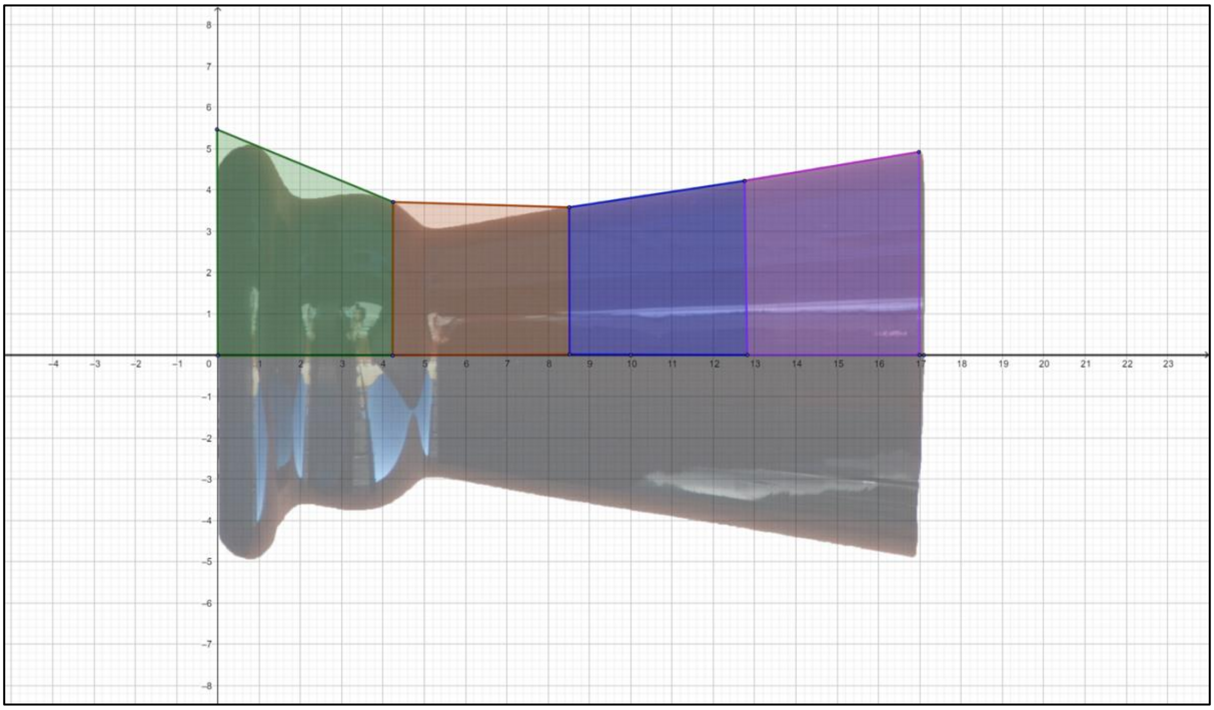
This formula can be derived from the Riemann sum which enables, among other applications, to approximate the area between continuous function and x-axis in the interval [a,b] by adding the areas of rectangles of equal width under the curve. As the number of these rectangles approaches infinity, they become infinitesimally thin and the Riemann integral is obtained which enables to calculate the real area under the curve in this interval. Then the area contained by the curve and x-axis in the interval [a, b] can be rotated by 2π radians around the x-axis to obtain the volume of solid of revolution. It should be noted that from the perspective of each infinitesimally thin cylinder which adds up to the volume of revolution, the slope of the curve is not taken into account because it has a negligible effect.

However, when it comes to calculating the lateral surface area of solid of revolution the slope of the curve has a significant effect on the final result, that is why frustum of a cone, instead of a cylinder must be introduced.

Each frustum is created by taking a trapezoid, with the same width Δx for all trapezoids, and rotating it around the x-axis. Down below in, Figure 2. I included the superimposed picture of vase with 4 trapezoids of the same width, Δx and different colors modeled onto the object. If they were rotated 2π times around the x-axis, 4 frustums would be created. It is clearly visible that the number of frustums is too small since the solid produced would merely imitate the vase. Therefore, infinite number of frustums should be used in calculation of the surface area.

Note: All the graphs in the work were produced in *Geogebra* (Hohenwarter & et al., 2016).

Figure 1. Vase with four 2D plotted frustums.



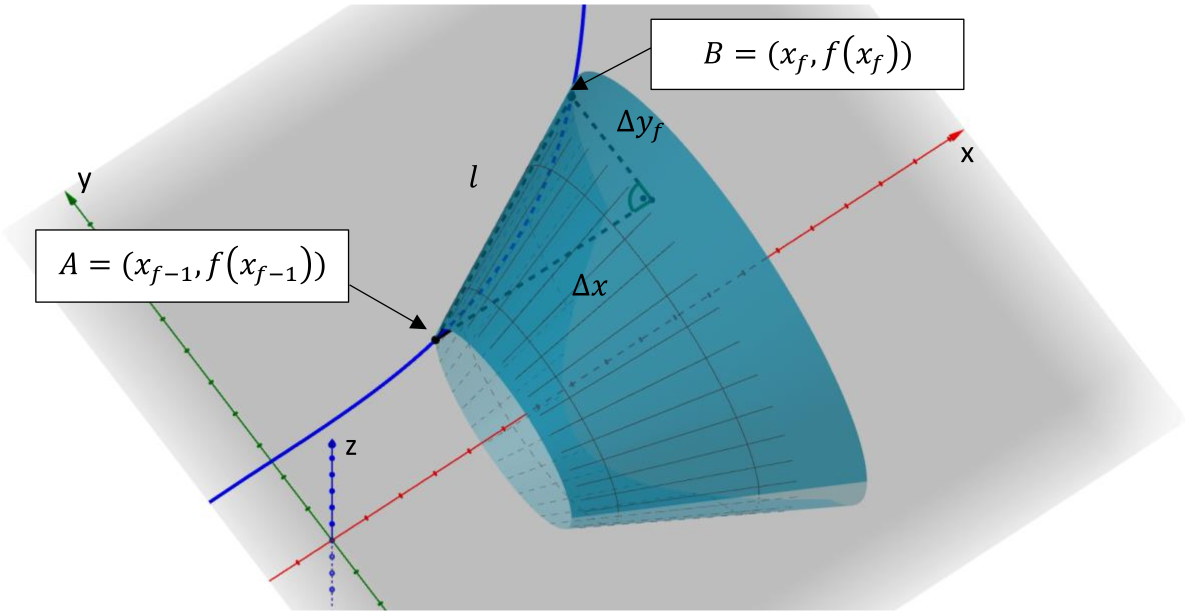
I will start deriving the formula for the lateral surface area of solid of revolution with the formula for lateral surface area of a frustum of a cone, A_f which is:

$$A_f = \pi(R + r)l$$

Where: R is the radius of the bottom base, r is the radius of top base, and l is the slant height.

In Figure 3. I included frustum modeled on interval $[x_{f-1}, x_f]$ of the continuous function produced by rotating the trapezoid described within points A and B, about the x-axis. Where points A and B lie on $f(x)$. It was performed to apply the formula for lateral surface area of frustum onto the function. I denote the lateral surface area of this frustum as A_b.

Figure 3. Frustum modelled onto the graph of continuous function.



Variables R and r are then substituted with corresponding y values on the graph, that is $f(x_f)$ and $f(x_{f-1})$, respectively. As a result, we get A_b:

$$A_b = \pi\big(f(x_f) + f(x_{f-1})\big)l$$

Where: l is slant height, Δx is the height of frustum Δy_f is the variable displayed in Figure 3. which changes depending on where on the graph, frustum is formed. In order to express slant height in terms of Δx and Δy_f, the Pythagorean Theorem is used:

$$l^2 = \Delta x^2 + \Delta {y_f}^2$$

Then, take the square root of both sides. Since the l is a length, only positive root is considered:

$$l = \sqrt{\Delta x^2 + \Delta {y_f}^2}$$

Factor out and take the square root of Δx:

$$l = \sqrt{\Delta x^2\left(1+\frac{\Delta {y_f}^2}{\Delta x^2}\right)}$$

$$= \sqrt{1+\left(\frac{\Delta y_f}{\Delta x}\right)^2}\,\Delta x$$

The mean value theorem states that "if f is continuous over the closed interval $[a, b]$ and differentiable over the open interval (a, b), then there exists a point $c \in (a, b)$ such that the tangent line to the graph of f at c is parallel to the secant line connecting $(a, f(a))$ and $(b, f(b))$" (OpenStaxCollege, 2016). According to this theorem, tangent and secant line are parallel, therefore their gradients are equal. It can be applied in the expression for slant height by substituting the gradient of secant line, that is slant height with the gradient of tangent to point x_a:

$$l = \sqrt{1+(f'(x_a))^2}\Delta x$$

The implication of applying the mean value theorem is that only the continuous functions in the interval [a, b] and differentiable over the interval (a, b) will satisfy the requirements to be inserted into the future formula for surface of revolution.

As such, the expression for slant height of frustum of cone can be substituted into the formula for A_b:

$$A_b = \pi\big(f(x_f)+f(x_{f-1})\big)\sqrt{1+(f'(x_a))^2}\Delta x$$

In order to link $f(x_f)$ and $f(x_{f-1})$ together I will apply the intermediate value theorem. It states that if $f(x)$ is continuous over the closed interval $[a, b]$, then there is point x_b, such that $a \leq x_b \leq b$ and $f(x_b) = y_b$. Where $y_b \in [f(a), f(b)]$ (Bazett, 2017). According to the intermediate value theorem, as long as the function $f(x)$ is continuous, there is a point x_b such that:

$$f(x_b) = \frac{1}{2}[f(x_f) + f(x_{f-1})]$$

By simple algebraic transformation:

$$f(x_f) + f(x_{f-1}) = 2f(x_b)$$

By substitution it can be introduced into the previous formula:

$$A_b = 2\pi f(x_b)\sqrt{1 + (f'(x_a))^2}\Delta x$$

As a result, from the sum of surface areas of frustums the approximate lateral surface area of the solid of revolution, A can be calculated:

$$A \approx \sum_{i=1}^{n} 2\pi f(x_b)\sqrt{1 + (f'(x_a))^2}\Delta x$$

As $n \to \infty$, the approximation becomes more and more similar to the real surface area of the solid of revolution. Since I recognized it as a Riemann sum, it can be expressed and a definite integral:

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi f(x_b)\sqrt{1 + \left(f'(x_a)\right)^2}\Delta x = \int_a^b 2\pi f(x_b)\sqrt{1 + \left(f'(x_a)\right)^2}\, dx$$

Since $f(x)$ is continuous, as $n \to \infty$, Δx becomes infinitesimally small, thus $f(x_b) = f(x)$ and s $f(x_a) = f(x)$. Therefore:

$$A = \int_a^b 2\pi f(x)\sqrt{1 + \left(f'(x)\right)^2}\, dx$$

3. Graphing the vase

I have already derived the formula for the lateral surface area of solid of revolution. To apply it and calculate the surface area of the vase, I have to create functions that will graph half of the vase.

Firstly, I zoomed in the vase with the lens and took the photo from a relatively long distance to reduce the effect of perspective. Then I measured the height of the vase by putting it upright on the table and using a ruler. To measure the circumference, I used inelastic string and coiled it around the bottom of the base. However, I noticed that the string kept sliding off and the measurement would be inaccurate. Therefore, I decided to measure the circumference at point

„G”, labelled in the Figure 4. since it is a narrowing in the structure and the string do not slide. The last step involved measuring the height at which point „G” occurs. Results are presented in Table 1.

Table 1. Measurements of the vase.

Height (cm)	Circumference at point G (cm)	Height of point G (cm)
17.10	19.00	5.00

Since the vase is a solid of revolution, in order to obtain the radius at point G, the formula for the circumference of circle was used:

$$Circumference\ of\ circle = 2\pi r$$

$$r = \frac{Circumference\ of\ circle}{2\pi}$$

Now, the radius can be calculated:

$$r = \frac{19.00}{2\pi} \approx 3.024\ (4\ s.f.)$$

In order to position the vase accurately on the graph, points from height and radius data were produced and plotted onto the graph. Their coordinates are presented in Table 2.

Table 2. Coordinates of G and J.

Point \ Coordinates	x	y
G	5.00	3.024
J	17.10	0

The opacity of vase image was reduced in order to make the grid behind the image visible. Then the picture was scaled and shifted so that it most accurately fit points „G” and „J”. As a result, it enabled creating other points digitally, to reduce the margin of human error, with coordinates in centimeters.

It was a dilemma for me how to describe the vase using the functions. On one hand, I could produce a single polynomial function describing the whole half-contour of the vase. On the other hand, I could divide the vase into regions and find the functions fitting into the corresponding regions. After some research, I learned that even though, the higher number of data points used to produce higher-degree polynomial ensures that the function passes perfectly through them, it does not ensure that the function between these data points will resemble the shape it is supposed to describe (Branden & Weisstein, n.d.). The first argument for using distinct functions is the fact that the part of the vase is a frustum and it is possible to accurately calculate its surface area using a linear function. An argument against producing single polynomial function is that it would involve many points, and the calculation would be a lot more complex where the accuracy of fit would not be ensured and the potential risk of human error involved in calculations would be higher. To conclude, I decided to divide the vase into regions and describe those partial shapes using different functions.

In order to accurately describe the vase in terms of mathematical functions, I have created total of 9 points. Their location was determined so that they would act as potential minimum / maximum points or points of inflexion. Their x-coordinates and y-coordinates are given in the Table 3. and their positions on the graph are depicted in Figure 4. I approximated that three functions going through these points will give an accurate outline of the vase. Then I highlighted their domains using three colors: green, orange and violet.

Figure 4. Points plotted onto the graph with superimposed image of the vase and three domains indicated by different colors.

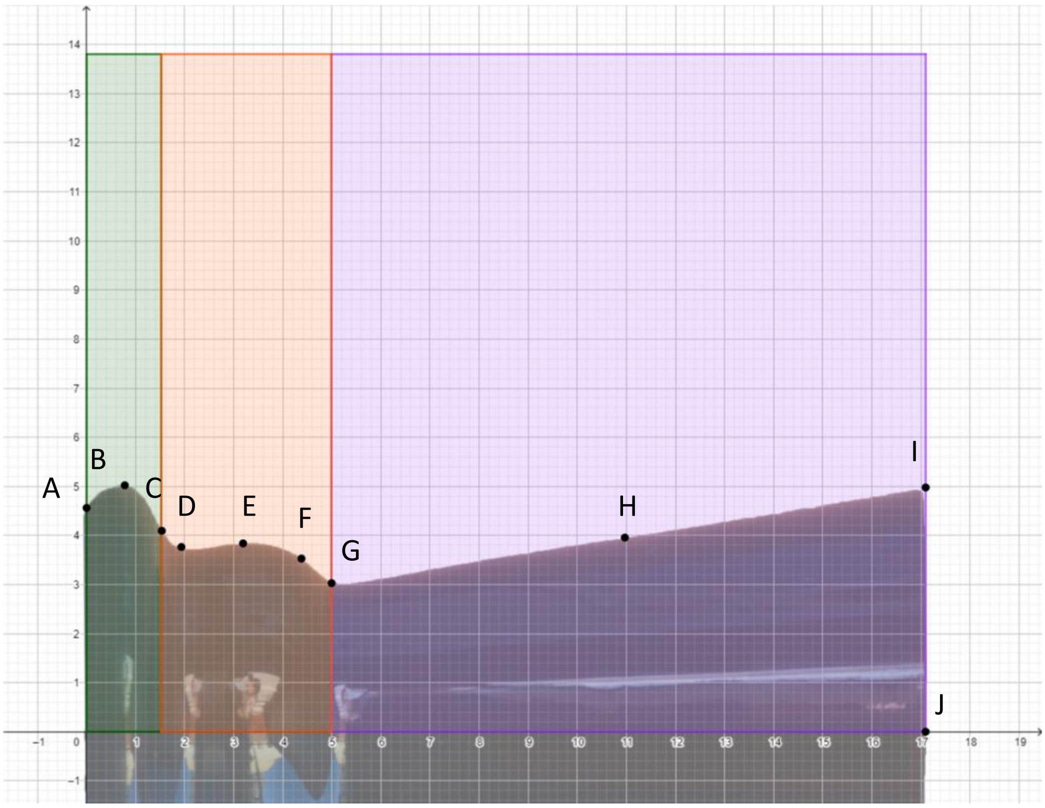


Table 3. Coordinates of ten points plotted onto the graph.

Point / Coordinates	A	B	C	D	E	F	G	H	I
x	0	0.679	1.53	1.93	3.19	4.38	5.00	11.0	17.1
y	4.560	5.00	4.09	3.76	3.84	3.53	3.024	3.96	4.98

4. Determining functions

Based on the Figure 4. I evaluate that the first two functions in green and orange regions will be polynomial. Interpolation is a method for determining function of x based on the known values of the function (The Editors of Encyclopaedia Britannica, 2016). In order to find these two functions, I will utilize Lagrange's Interpolation Formula which can be used even if the points are at unequal intervals from each other. The formula to calculate a polynomial $g(x)$ is as follows:

$$g(x) = \frac{(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-x_2)(x_1-x_3)\dots(x_1-x_n)}y_1 + \frac{(x-x_1)(x-x_3)\dots(x-x_n)}{(x_2-x_1)(x_2-x_3)\dots(x_2-x_n)}y_2 + \cdots + \frac{(x-x_1)(x-x_2)\dots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\dots(x_n-x_{n-1})}y_n$$

Where: Degree of function $\leq (n-1)$ and $g(x)$ passes through n points: $(x_1, y_1 = f(x_1)), (x_2, y_2 = f(x_2)), \dots, (x_n, y_n = f(x_n))$ (Branden & Weisstein, n.d.).

Note: The calculator available at website: *https://planetcalc.com/8680/* was used to compute the results.

The function in the violet region produced from points H and I will be calculated using gradient formula and equation of a straight line, which are respectively:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = m(x - x_1)$$

4.1. Bottom section

In this part I will find function for the bottom section of the vase with the domain indicated by green color in Figure 4. Based on the shape of the curve of the vase it appears that it is a concave-down quadratic function with maximum point around point B (0.679, 5.00). Even though if one more point was determined and the cubic function were created, it would not guarantee a better fit with the shape as I outlined previously. Also, the calculation would be longer and the probability of human error would increase. I denote the function of the bottom section as $g_I(x)$. I will calculate $g_I(x)$ using Lagrange's Interpolation Formula from three data points shown below:

Table 4. Coordinates of points: A, B and C.

Point \ Coordinates	x	y
A	0	4.560
B	0.679	5.00

C	1.53	4.09

Therefore when function $g_1(x)$ passes through 3 points (0, 4.560), (0.679, 5.00), (1.53, 4.09), its formula is:

$$g_I(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-0.679)(x-1.53)}{(0-0.679)(0-1.53)} \times 4.560 + \frac{(x-0)(x-1.53)}{(0.679-0)(0.679-1.53)} \times 5.00$$

$$+ \frac{(x-0)(x-0.679)}{(1.53-0)(1.53-0.679)} \times 4.09$$

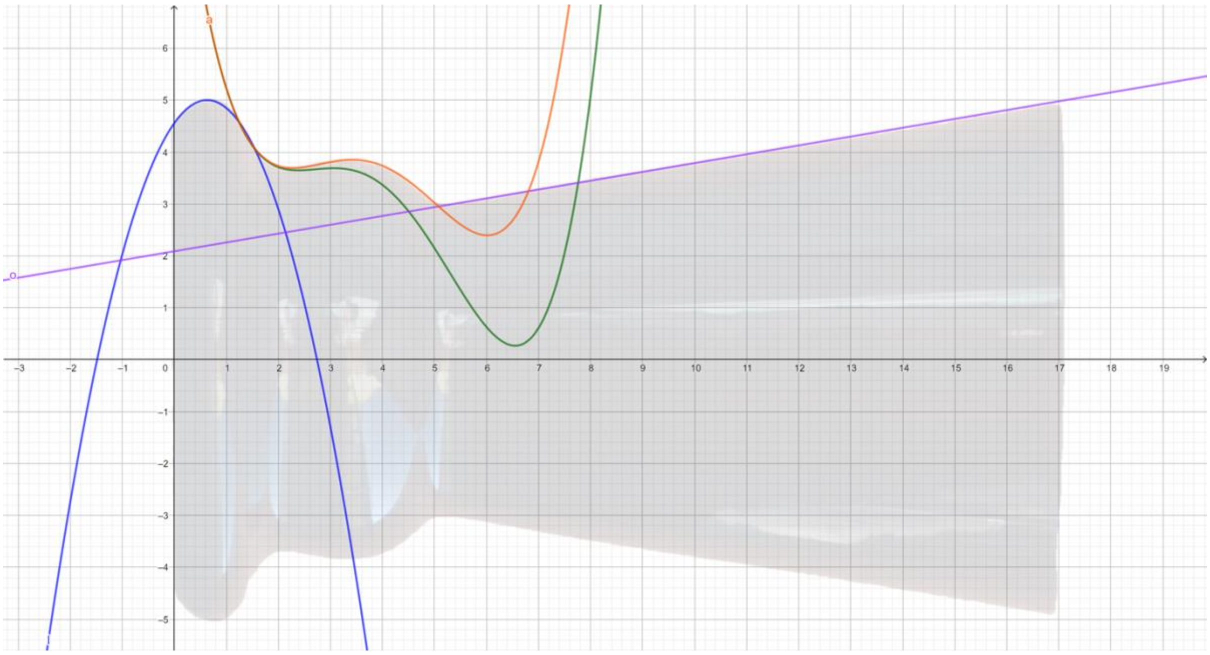
$$= -\frac{99233000}{88407837}x^2 + \frac{124668527}{88407837}x + \frac{114}{25}$$

$$= -1.12x^2 + 1.41x + 4.56\ (2\ d.p.)$$

Therefore, the function describing the bottom section of the vase is defined as: $-1.12x^2 + 1.41x + 4.56$

I plotted function $g_1(x)$ in blue color onto the superimposed picture of vase and the result is visible in Figure 5. Even though a miniscule part of the vase is above the function and will not be included in calculation of the surface area of the vase, it is insignificant and can be omitted because the overall fit is good.

Figure 5. Graph with superimposed picture of the vase and all functions



4.2. Middle section

In this part I will find function describing the bump in the shape of the vase that occurs on the right to the bottom section and should be restricted by the domain indicated by orange color in Figure 4. Arrangement of points resembles quartic function. Therefore, I have chosen total of 5 points to put them in the Lagrange's Interpolation Formula, according to the restriction, that: degree of polynomial $\leq (n-1)$. If I have chosen smaller number of them, the resulting function would less accurately describe the shape because the significant points on the shape would not be included.

I denotes the function representing middle section as $g_2(x)$ and in order to find its equation I will use the following points:

Table 5. Coordinates of points: C, D, E, F, G.

Point \ Coordinates	x	y
C	1.53	4.09
D	1.93	3.76

E	3.19	3.84
F	4.38	3.53
G	5.00	3.024

Therefore, when function $g_2(x)$ passes through 5 points (1.53, 4.09), (1.93, 3.76), (3.19, 3.84), (4.38, 3.53), (5.00, 3.024) its formula is:

$$g_2(x) = \frac{(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)}y_1$$

$$+ \frac{(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)}y_2 + \cdots$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)}y_5$$

$$= \frac{(x-1.93)(x-3.19)(x-4.38)(x-5.00)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)} \times 4.09$$

$$+ \frac{(x-1.53)(x-3.19)(x-4.38)(x-5.00)}{(1.93-1.53)(1.93-3.19)(1.93-4.38)(1.93-5.00)} \times 3.76 + \cdots$$

$$+ \frac{(x-1.53)(x-1.93)(x-3.19)(x-4.38)}{(5.00-1.53)(5.00-1.93)(5.00-3.19)(5.00-4.39)} \times 3.024$$

$$= \frac{2527400125592500}{49468115162426877}x^4 - \frac{4384757980934250}{5496457240269653}x^3$$

$$+ \frac{849984096852554171}{197872460649707508}x^2 - \frac{105152960511486608 99}{10992914480539306 00}x$$

$$+ \frac{3636341414946946141}{323321014133509000} =$$

$$= 0.05x^4 - 0.80x^3 + 4.30x^2 - 9.57x + 11.25\ (2\ d.p.)$$

As such, the function representing the middle section of the vase is defined as: $0.05x^4 - 0.80x^3 + 4.30x^2 - 9.57x + 11.25$. It is stated with two decimal places due to the uncertainty of the measurement.

Then, I plotted function $g_2(x)$ in green color onto the superimposed picture of the vase and the result is presented in Figure 5. As it can be seen the function does not describe the shape accurately and major part of the vase is above the function. As such I wondered whether rounding the function to more decimal places will improve the fit. I started with 3 decimal places and noticed that each time rounding to one more decimal place gives better fit. I decided to round the function to 5 decimal places, because each next additional decimal place changes the curve very slightly. The improved function is presented in orange color in Figure 5. and it is clearly visible that the fit is far more precise. Therefore, the function describing the middle section is:

$$g_2(x) = \frac{2527400125592500}{49468115162426877}x^4 - \frac{4384757980934250}{5496457240269653}x^3 + \frac{849984096852554171}{197872460649707508}x^2 - \frac{10515296051148660899}{1099291448053930600}x + \frac{3636341414946946141}{323321014133509000}$$

$$= 0.05109x^4 - 0.79774x^3 + 4.29562x^2 - 9.56552x + 11.24685\ (5\ d.p.)$$

4.3. Top section

It is clearly visible in Figure 4. that the top section of the vase with the domain illustrated by purple color can be described using linear function with constant gradient throughout, therefore only two points are needed to describe it. I denoted the linear function as $g_3(x)$ and used the following points to calculate it:

Table 6. Coordinates of points: H and I

Point \ Coordinates	x	y
H	11.0	3.96
I	17.1	4.98

I started by calculating the gradient of function $g_3(x)$:

$$m = \frac{4.98 - 3.96}{17.1 - 11.0}$$

$$= 0.17\ (2\ d.p.)$$

Then I substituted the coordinates of point H into the formula for the equation of a straight line:

$$y - y_1 = m(x - x_1)$$

$$g_3(x) - 3.96 = 0.17(x - 11.0)$$

$$g_3(x) = 0.17x + 2.09\ (2\ d.p.)$$

Therefore, the function describing the top section of the vase is defined as: $0.17x + 2.09$

In order to be certain of the good fit of the function $g_3(x)$, I plotted it in purple color onto the picture of vase and the result is presented in Figure 5. The fit is almost perfect and the minimal number of white pixels in the right part of the vase which are visible under the curve may be due to not ideal removal of the background in *Adobe Photoshop*. Therefore, I perceive the function describing the top section as an accurate and good representation.

4.4. Improvements of the final function

I noticed that when domains visible in Figure 4 are applied, there is a sharp transition between functions $g_2(x)$ and $g_3(x)$ what I depicted in Figure 6 with label "Before". In this part, the final function (describing the entire half of the vase) would not be continuous and if the volume of revolution was created there would be a sharp groove rather than smooth curve of the vase. Therefore, I have decided to improve the final function by changing the domains and will establish the new ones based on the point of intersection between functions $g_2(x)$ and $g_3(x)$.

To find the point of intersection, two functions are equated:

$$0.05109x^4 - 0.79774x^3 + 4.29562x^2 - 9.56552x + 11.24685 = 0.17x + 2.09$$

$$0.05109x^4 - 0.79774x^3 + 4.29562x^2 - 9.73552x + 9.15685 = 0$$

Then the roots are found:

$$x = 5.08 \text{ or } x = 6.81 \text{ (2 d.p. from GDC)}$$

It is visible on Figure 5. that we consider intersection at lower x value, therefore $x = 5.08$

Figure 6. Comparison of part under investigation before and after change in domain

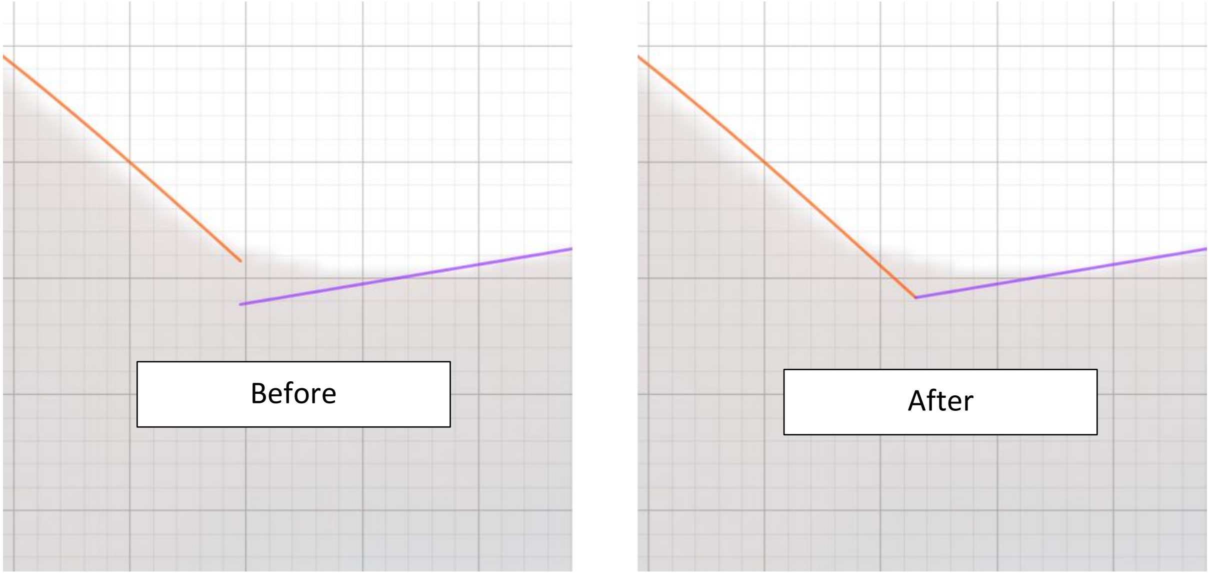


Figure 6. allows to compare the area under investigation between functions $g_2(x)$ and $g_3(x)$ and it is clearly visible that the domain change, labelled "After" has been beneficial, since the smaller part of the vase is above the functions. Moreover, two functions merge smoothly.

To include all the functions and their domains, the function $f(x)$ describing the whole vase is introduced:

$$f(x) = \begin{cases} -1.12x^2 + 1.41x + 4.56 & 0 \leq x \leq 1.53 \\ 0.05109x^4 - 0.79774x^3 + 4.29562x^2 - 9.56552x + 11.24685 & 1.53 \leq x \leq 5.08 \\ 0.17x + 2.09 & 5.08 \leq x \leq 17.1 \end{cases}$$

5. Calculations of the lateral surface area of the vase

To find the lateral surface area of the vase, the formula for surface area of solid of revolution was used. The underside of the vase is not incorporated into calculations because it would not be painted.

5.1. Bottom section

The function of the bottom section is: $g_1(x) = -1.12x^2 + 1.41x + 4.56$ and it is rotated around x-axis in the interval [0, 1.53].

Firstly, it is essential to find the derivative of that function:

$$g_1'(x) = -2.24x + 1.41\ (2\ d.p.)$$

Now, everything can be substituted into formula for A and obtain the surface area of the bottom section, denoted as A_{bot}:

$$A_{bot} = \int_0^{1.53} 2\pi \times (-1.12x^2 + 1.41x + 4.56) \times \sqrt{1 + (-2.24x + 1.41)^2}\, dx$$

$$= 63.19\ cm^2\ (2\ d.p.from\ GDC)$$

5.2. Middle section

The function of the middle section is: $g_2(x) = 0.05109x^4 - 0.79774x^3 + 4.29562x^2 - 9.56552x + 11.24685$ and it is rotated around x-axis in the the interval [1.53, 5.08].

Similarly, derivative of that functions has to be found:

$$g_2'(x) = 0.20436x^3 - 2.39322x^2 + 8.59124x - 9.56552\ (5\ d.p.)$$

Now, everything can be substituted into formula and obtain the surface area of the middle section, denoted as A_{mid}:

$$A_{mid} = \int_{1.53}^{5.08} 2\pi(0.05109x^4 - 0.79774x^3 + 4.29562x^2 - 9.56552x + 11.24685\ ) \times \sqrt{1 + (0.20436x^3 - 2.39322x^2 + 8.59124x - 9.56552)^2}\, dx$$

$$= 91.38cm^2\ (2\ d.p.from\ GDC)$$

5.3. Top section

The function of the top section is: $g_3(x) = 0.17x + 2.09$ and it is rotated around x-axis in the interval [5.08, 17.1].

Gradient of linear function is constant, and as such it is equal to:

$$g_3{}'(x) = 0.17$$

Now, everything can be substituted into formula and obtain the surface area of the top section, denoted as A_{top}:

$$A_{top} = \int_{5.08}^{17.1} 2\pi \times (0.17x + 2.09) \times \sqrt{1 + 0.17^2}\, dx$$

$$= 304.54\ cm^2\ (2\ d.p. from\ GDC)$$

5.4. Total outer surface area of the vase

To find the total outer surface area of the vase, all the previously calculated surface area values are added, they include: bottom, middle, and top sections.

$$A_{total} = A_{bot} + A_{mid} + A_{top} = 63.19\ cm^2 + 91.38\ cm^2 + 304.54\ \ cm^2$$

$$= 459.11 cm^2\ (2\ d.p.)$$

6. Calculations of the cost of repainting

The paint coverage I assumed due to similarity to the nature of the acrylic enamel paint is 9 m^2/l. The price of the paint is 45.99£ for 18 paint tubes, each containing 5ml of paint. Firstly, I will determine how much paint (in litres) will be needed to cover the lateral surface of the vase:

$$9 \times 10^4 cm^2 - l$$

$$459.11 cm^2 - x$$

Therefore, the amount of paint needed is:

$$x = 5.10 \times 10^{-3} l\ (2\ d.p.)$$

Now, I will calculate how much this amount of paint would cost me:

$$45.99£ - 0.090\text{l}$$

$$x - 5.10 \times 10^{-3} l$$

Therefore, the cost of paining the lateral surface of the vase would be:

$$x = 2.61£\ (2\ d.p.)$$

7. Conclusion

The aim of my investigation has been to determine the lateral surface of the vase to calculate the cost of painting it with acrylic enamel and decide whether it is a viable option. In order to do that I extended my knowledge in calculus and derived the formula for the lateral surface area of the solid of revolution. Also, based on the initial measurements of the vase, I plotted points onto the graph to model the vase using function. Then I applied the derived formula and calculated that the lateral surface of the vase is $459.11cm^2$.

Since the vase cost me 7.5£ and the potential cost of painting is 2.61£, I think that it is a viable option to enhance its appearance and extend its usage for approximately 35% of the initial price.

Beyond estimating the price of paining the vase, I established the method to calculate the lateral surface area of majority of solids of revolution with relatively high accuracy. It may be useful in the industry sector for evaluating the costs of painting decorations on mass scale. In the next section I will outline which shapes cannot be used. Also, I presented how the function describing the desired shape can be improved.

Moreover, I learnt the futuristic methods of visualizing mathematical functions and three-dimensional solids using *Geogebra.*

8. Evaluation

Strength associated with my work are plotting the new data points onto superimposed picture of the vase digitally so that the human error was decreased and accuracy was increased. Also, using several functions instead of one to graph the shape of the vase ensured very good fit with the actual shape, what is visible in Figure 5.

One limitation concerning my exploration is the uncertainty of the ruler used to measure parameters of the vase. Since its smallest division is 0.1 cm, the uncertainty is 0.05cm. Thus, the measurements had to be rounded to 2 decimal places and it ultimately resulted in relatively small accuracy of calculated lateral surface area of the vase. In order to improve it, more precise ruler, such as one with smallest division equal to 0.01 cm. Also, since the mean value theorem and intermediate value theorem have been used in deriving the formula for lateral surface area of solid of revolution, the function that is inserted into equation must be continuous in the interval [a, b] and differentiable over the interval (a, b). That implies that it cannot contain any discontinuities or undifferentiable intervals, for example when there is “kink” in the shape. Another room for error that could have contributed to inappropriately chosen points around the shape of the vase is human error associated with scaling and positioning the image of vase after initial points (based on measurements) have been plotted. Also, the long calculations involved in applying Lagrange’s Interpolation Formula offer high probability of human error when inserting the value into GDC, therefore it would be beneficial to write a code that would automatically calculate the equation of polynomial function. It would shorten the time needed for calculations, reduce the human error and facilitate calculating higher-degree polynomials from higher number of data points.

9. Future suggestions

This investigation could be extended by calculating the volume of glass used to produce the entire vase. Then the result could be evaluated by using the water displacement method in which the vase would be completely submersed in water in the bucket with volume division. The volume of displaced water would represent the real volume of the vase.

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